Dynamic Demand and Sequential Monopoly: A Model of Endogenous Screening

V. Bhaskar
UT Austin

Nikita Roketskiy
UCL

January 18, 2019

Abstract

We analyze a model with identical consumers, whose demand depends on the history of past purchases, and short-lived monopolists, that use nonlinear pricing. We focus on the case in which consumers' purchase histories are private. In any equilibrium, all sellers offer a large variety of bundle sizes paired with quantity discounts and taste heterogeneity arises endogenously due to differences in the past consumers' choices. We show that consumers' privacy is undesirable only for a seller that faces a homogeneous client base and has the first-mover advantage; it benefits all other market participants.

Keywords: dynamic demand, endogenous screening, nonlinear pricing.
JEL Codes: D11, D43, L13

[PRELIMINARY AND INCOMPLETE. PLEASE DO NOT CIRCULATE.]

1 Introduction

Do supermarkets encourage wasteful purchases of goods with a limited shelf-life, such as fruit or vegetables?[1] Do fast-food restaurants induce over-consumption? An important feature of consumers' preferences in these markets is inter-temporal substitutability: if a patron has a heavy lunch, she is less hungry at dinner. If a

[1]The "super-size-me" phenomenon and the obesity epidemic is indicative, as is the documented evidence on the waste of perishables after spending several days in the fridge. While intrinsic consumer preferences are surely important, the present paper highlights the role of a different mechanism.

consumer buys a two-for-one deal on salads or ready meals at the supermarket, he is less likely to buy similar goods when stopping at the local store. In these instances, purchases at different dates are substitutes. A different set of examples concerns habit formation or goods where taste can gradually develop, and where the concern is that there might be a tendency to under-consumption. A student who frequents jazz concerts is more likely to enjoy them later in life, so that consumptions across dates are complements.

In both types of examples, purchases at different dates are often from different suppliers. For instance, in the context of grocery stores and supermarkets, multi-store shopping is a widespread and well-documented phenomenon: large share of consumers visit more than one store, often on different days.[2]

In this paper, we study optimal nonlinear pricing in the presence of either intertemporal substitutability or complementarity. Consumers' privacy plays an important role in our analysis. It has a profound effect on equilibrium per-unit price dispersion, social welfare and the allocation of surplus between the market participants. When consumers' history of purchases is unobservable by sellers, our model derives the relationship between the distribution of per-unit prices and the degree of intertemporal substitutability or complementarity.

We develop a model in which consumer's willingness to pay for a good depends on the past history of consumption, and in which a consumer, who shops with a supplier today, is unlikely to return tomorrow. To focus on the dynamic implications of endogenous choices, we assume that consumers are identical — i.e., any differences in taste only arise due to differences in past consumption. Furthermore, we assume that a supplier at any date has monopoly power — e.g., because of search frictions — so that the market is characterized by sequential monopoly. We assume that consumers have a quasilinear utility and that willingness to pay for consumption c_t depends on the past consumption c_{t-1}: $u(c_t, c_{t-1})$. Depending on a particular application, u is either submodular or supermodular.

We allow sellers to offer fully non-linear prices and analyze the nature of intertemporal competition. We consider the simplest version of our model, a two-period one. In a benchmark case, where past consumption is observable by the monopolist today, our intuitions are confirmed — the first period monopolist induces over-consumption relative to the efficient allocation when u is submodular, and under-consumption when u is supermodular.[3]

[2]Thomassen et al. (2017) document evidence of multi-stop shopping in the UK, and Fox et al. (2004) — in the U.S.

[3]This finding is reminiscent of the literature on long-term contracts, where payoffs and utility are stationary. Here, contracts are short-term, but future utility depends on current consumption.

Our main focus however, is on the more realistic case where the consumer's past history is unobserved by the current seller. In such an environment, when a seller prices his goods, he has to take into account heterogeneity of consumers' tastes that arises due to differences in past consumption. This heterogeneity is affected by the pricing strategies of the seller's competitors. In this paper, we show, that even if consumers are ex ante identical, a large variety of quantities offered by sellers in equilibrium gives rise to persistent ex post taste heterogeneity.

We begin our analysis by showing that there cannot be a pure strategy equilibrium where consumption is deterministic, both when u is submodular and when it is supermodular. More generally, if the seller offers two options to the consumer in the second period, he cannot extract the full difference in value between these two options in equilibrium—the consumer is always able to retain some surplus. Therefore, consumers must accumulate private information in *any* equilibrium.

We, therefore, look for mixed strategy equilibria, and we show that there exists a continuum of equilibria, but all of them feature the same consumption paths and differ only in the distribution of surplus between the consumer and the sellers. In these equilibria, the first period monopolist offers a large menu, which ranges between the efficient quantity and that chosen in the observable case. The consumer, who is indifferent between all bundles in the menu, chooses an item according to a continuous distribution with full support. This induces an endogenous screening problem in the second period, since the consumer has private information about his past consumption. We find that the consumer and the second-period seller benefit from unobservability, whereas the first-period seller loses, as compared to the observable consumption benchmark.

2 Related literature

Our model bears considerable formal similarity with models of common agency (Bernheim and Whinston (1986); Martimort and Stole (2002)) – the principals in these models correspond to our sellers, and the agent to the consumer. A key difference is that interaction (and competition between sellers) is sequential in our context, whereas in common agency models, the principals compete simultaneously. Thus sequential rationality plays a critical part in our analysis – when a consumer receives an offer from a seller today, she does not have the option of revising her purchases yesterday. Whereas common agency models have a plethora of equilibria and use refinements such as truthfulness to single out a few, we find that equilibrium is essentialy unique, at least in the two-period setting.

Our paper also relates to the literature on long-term bilateral contracts in a multilateral environment, including Diamond and Maskin (1979) and Aghion and Bolton (1987). In contrast with this literature, our contracts are static, and the dynamics are induced by the agent's preferences.

[seller's commitment∼observability of the past]

A key aspect of our analysis is the endogenous screening problem that arises, due to the fact that the agent's choice is payoff-relevant as well as private information. This is reminiscent of the work on static moral hazard with renegotiation – Fudenberg and Tirole (1990) and Ma (1991). More recent work where types are endogenous includes González (2004), Calzolari and Pavan (2006) and Netzer and Scheuer (2010).

Finally, our work relates to an emerging literature on the pricing of storable goods, theoretical as well as empirical. This includes Hong et al. (2002), Hendel and Nevo (2006a), Hendel and Nevo (2006b), Ariga et al. (2001) and Hendel et al. (2014).

Cole and Kocherlakota (2001), Holmstrom and Milgrom (1987), Villas-Boas (1999), Villas-Boas (2004), and Villas-Boas (2006) are also related.

3 The model

The consumer, who lives for two periods, visits seller 1 in the first period and seller 2 in the second period. Her utility is

$$u(y,x) - p - q,$$

where x and y is consumption in the first and second period respectively, and p and q are the payments made to sellers 1 and 2 respectively. The value of consumption in the second period depends on the level of consumption in the first period. We assume that u is strictly increasing, strictly concave and twice continuously differentiable. We consider two alternative assumptions:

A1: $u(y,x)$ is strictly supermodular; or

A2: $u(y,x)$ is strictly submodular.

The sellers maximize their profit by offering the consumer a menu of nonlinearly priced bundles. For example, seller 1 chooses a lower semi-continuous function $p : \mathbb{R}_+ \to \mathbb{R}_+$, where $p(x)$ is a price that this seller charges for a bundle of size x. Long-term contracts are not feasible — an exchange between the seller and the consumer happens within one period. We assume that the sellers produce the good at a constant marginal cost k.

3.1 Benchmarks

Before we proceed with the analysis of our model, we consider two useful benchmarks: social planner's problem, and a model in which the past history is observable by the second-period seller.

The socially efficient level of consumption (x^*, y^*) is defined as follows. Define $y^*(x)$ as the value of y that solves $u_1(y,x) = k$, if this equation has a positive solution, and zero otherwise. Since u_1 is strictly decreasing in y, there is a unique value $y^*(x)$. Let x^* be the value of x that solves $u_2(y^*(x), x) = k$, and let $y^* := y^*(x^*)$. Since u is strictly concave, there is unique solution, so that the socially efficient level of consumption (x^*, y^*) is unique.

Consider the model, in which the first-period consumption is perfectly observed by the second-period seller. In this case, seller 2 chooses y that maximizes $u(y,x) - ky$, and sets a price that makes the consumer indifferent between accepting y and her outside option of consuming zero units of good. Thus, seller 2 chooses $y = y^*(x)$, and sets $q = u(y^*(x), x) - u(0, x)$, and the consumer's second period payoff (net of sunk payments made to seller 1) $u(y^*(x), x) - q$ equals $u(0,x)$. The consumer's utility from consuming x equals

$$u(0,x) - p.$$

If the consumer does not buy from the first period monopolist, her overall payoff equals $u(0,0)$ (since her second period continuation payoff after any x always equals $u(0,x)$). Since the first period seller optimally sets p so that

$$p(x) = u(0,x) - u(0,0),$$

the bundle that maximizes the first period profits when consumption is observable, x^o, satisfies

$$u_2(0, x^o) = k.$$

We shall assume throughout:

A3: $y^o := y^*(x^o) > 0$.

This assumption ensures that the first seller cannot serve the customer alone and the second seller plays a meaningful role in this market.

If u is strictly submodular, then notice that $x^o > x^*$ so that the first period monopolist induces excessive consumption relative to the first best. If u is strictly supermodular, then $x^o < x^*$, then the first period seller induces underconsumption relative to the first best. In either case, the reason is that the first period seller takes into account how he affects the consumer's second period outside option, $u(0,x)$, rather than the consumer's utility associated with his actual consumption, $u(y^o, x)$.

4 Unobserved past

In this section, we consider the main specification of our model in which seller 2 can observe neither the offer made by seller 1 nor the consumer's choice in the first period. Our first result is that there does not exist a pure strategy equilibrium in this case, either in the supermodular or the submodular case.

In a pure strategy equilibrium, seller 2 correctly conjectures consumer's past consumption x and best-replies to this conjecture by pricing the good accordingly (recall, that the past consumption is relevant for the valuation of good in the second period). Since seller 2 is the monopolist, he sets the price of a bundle chosen by the consumer to make the consumer indifferent between the inside and the outside option.

If $u(y, x)$ is submodular, seller 1 has a profitable deviation. Instead of selling x, he can offer $\tilde{x}$ that is slightly larger than x. If the consumer increases his consumption in the first period, he will be less hungry in period 1 — i.e., his willingness to pay for the good in period 2 will be lower — and he will choose a smaller bundle in the second period. In particular, if a singleton menu is offered in period 2, the consumer will choose the outside option of zero. Notice, that seller 2 will not be able to react and reduce his prices, because he cannot observe the past consumption.

As a result of this deviation, the joint surplus of seller 1 and the consumer will increase, because seller 2 will receive a smaller revenue. The only thing left to do for seller 1 is to divide this surplus between the consumer and himself so that the consumer accepts the new bundle $\tilde{x}$.

Put differently, no matter what seller 2 conjectures, seller 1 will always try to oversell compared to seller 1's beliefs and take away a fraction of seller 2's profits through the consumer's intertemporal substitution. Similar argument applies for the case of supermodular $u(y, x)$; the difference is that seller 1 will undersell rather than oversell.

Actually, this observation is more general. The consumer cannot be indifferent between two different bundles offered by seller 2 in equilibrium.

Lemma 1. *Suppose that the consumer buys $\hat{x}$ in the first period and chooses one of the two available options, (q_1, y_1) and (q_2, y_2) that satisfy the following:*

(i) $y_1 < y_2$;

(ii) $u(y_1, x) - q_1 = u(y_2, x) - q_2$; *and*

(iii) $u_1(y_2, x) \geq k$.

Then one of the sellers has a profitable deviation.

Proof. We prove this lemma for the submodular $u(y, x)$. The proof for the supermodular $u(y, x)$ is similar.

The difference in profit between the two options that are offered in the second period are

$$q_2 - ky_2 - (q_1 - ky_1) = u(y_2, x) - u(y_1, x) - k(y_2 - y_1) > 0.$$

If the consumer chooses (q_1, y_1), seller 2 can reduce q_2 by arbitrarily small but positive amount and, thus, increase his profit.

If the consumer chooses (q_2, y_2), seller 1 has a profitable deviation. Indeed, if seller 1 offers $\hat{x} > x$ instead of x, the consumer will choose either (q_1, y_1) or another, even smaller bundle. Similarly, if seller 1 offers $\hat{x} < x$ instead of x, the consumer will choose either (q_2, y_2) or another, even larger bundle. Thus, the total payoff of the consumer and seller 1, $\Pi(x)$, is bounded from below by

$$\Pi(x) \geq \hat{\Pi}(x) := \begin{cases} u(y_1, x) - q_1 - kx & \text{, if } x \geq \hat{x} \\ u(y_2, x) - q_2 - kx & \text{, if } x \leq \hat{x}. \end{cases}$$

and $\Pi(\hat{x}) = \hat{\Pi}(\hat{x}) = u(y_1, \hat{x}) - q_1 - k\hat{x}$. The function $\hat{\Pi}(x)$ is continuous and u is submodular, hence

$$\hat{\Pi}'(\hat{x}^+) - \hat{\Pi}'(\hat{x}^-) = u_2(y_1, \hat{x}) - u_2(y_2, \hat{x}) > 0.$$

Therefore $\hat{\Pi}(x)$ cannot achieve the maximum at $\hat{x}$, and so cannot $\Pi(x)$.

□

This lemma allows us to show our first result on non-existence of pure strategy equilibrium and it hints on how to construct the mixed strategy equilibria in this model. A crucial ingredient is the uniqueness of the consumer's optimal choice in the second period — a feature that naturally occurs when seller 2 offers the optimal screening menu.

Proposition 2. *If $u(y, x)$ is strictly submodular or strictly supermodular, and assumption A3 is satisfied, there does not exist an equilibrium where the consumption in period 1 is deterministic.*

Proof. If the consumption in period 1 is deterministic, say $\hat{x}$, the best reply by seller 2 is a singleton menu (q, y) such that

$$u(y, \hat{x}) - q = u(0, \hat{x})$$
$$u_1(y, \hat{x}) = k.$$

Since both $(0, 0)$ and (q, y) are available, conditions of Lemma 1 are satisfied, and, therefore, one of the two sellers has a profitable deviation. □

4.1 Endogenous screening

All equilibria in this model have several features in common. The first period consumption is random — seller 1 chooses an interval $[\underline{x}, \bar{x}]$ and offers a menu p such that the consumer never buys quantities outside of this interval. The menu p is a two-part tariff in which a per-unit charge equals marginal cost of production — i.e., if the consumer purchases quantity $x > \underline{x}$, she compensates the seller for the cost of producing $(x - \underline{x})$ extra units of good. The expected profit that seller 1 receives in equilibrium is $p(\underline{x}) - k\underline{x}$.

The consumer chooses the consumption in the first period randomly, according to a continuous distribution $F(x)$ with full support on $[\underline{x}, \bar{x}]$. Since the first period consumption is both a consumer's private information and relevant for consumer's willingness to pay for the second period bundles, seller 2 offers a screening menu q. It is convenient to think of this menu in the form of a direct mechanism $(\hat{y}(x), q(x))$. The allocation rule $\hat{y}(x)$ is always strictly monotone and this mechanism is equivalent to a menu $q(\hat{y}^{-1}(y))$ (if we restrict our attention to bundles that are chosen on equilibrium path).

In order to elicit the information about the past consumption, seller 2 leaves the consumer with some information rent $U(x)$. When the consumer makes a purchase in the first period he chooses both her current consumption and her future information rent. It is optimal for her to randomize because any decrease in the net value of the current consumption is compensated by an increase in information rent.

To summarize, the critical features for any equilibrium are as follows:

1. $U(x) - kx$ is constant for every $x \in [\underline{x}, \bar{x}]$. This ensures that the consumer and seller 1 are indifferent as to which element of $[\underline{x}, \bar{x}]$ the consumer chooses.

2. The induced distribution of the first period consumption, F, is such that seller 2 finds it optimal to offer $U(x)$ for each $x \in [\underline{x}, \bar{x}]$.

3. $\hat{y}(x)$ is strictly decreasing in the submodular case, and strictly increasing in the supermodular case.

4. Finally, the endpoints of the interval $[\underline{x}, \bar{x}]$ are pinned down by the characteristics of the solution to the monopoly screening problem. Since there is no distortion at the top, the second period consumption of the highest type — e.g., $\underline{x}$ in the submodular case — must be optimal given $\underline{x}$. Combined with point (1) above, this implies that $\underline{x} = x^*$, the first best level of consumption (when utility is supermodular, the highest type corresponds to $\bar{x}$ which must equal x^*). Since there is no informational rent at the bottom — e.g., for type $\bar{x}$ in

the submodular case — her consumption level must maximize the joint payoff of the consumer and seller 1 given that she takes the outside option 0 in the second period. This implies $\bar{x} = x^o$. Thus, the first period consumptions span the range between first best and the equilibrium consumption in the case when the past history is observable, while second period consumptions lie between 0 and the first best consumption, y^*.

4.2 Equilibrium characterization

In this section we formalize the ideas presented in Section 4.1. We begin with the characterization of the continuation payoff in the second period and then we provide conditions that pin down the distribution of consumption in the first period.

Proposition 2 has established that in any equilibrium, the first period consumption must be random. Let X denote the support of the equilibrium distribution of the first period consumption — X is a closed set, by definition. We shall also assume that every bundle in X is offered and chosen by the consumer.[4] Note that X cannot contain 0 — in this case, seller 1's profits must equal zero, and this cannot be optimal for seller 1 and the consumer.

Denote an incentive compatible direct mechanism offered by seller 2 in equilibrium by $(\hat{y}(x), q(x))_{x \in X}$. The information rent of the consumer after choosing bundle x in the first period is

$$U(x) := u(\hat{y}(x), x) - q(x).$$

The sum of the payoffs for the consumer and seller 1 if the former chooses x is

$$\Pi(x) := U(x) - kx.$$

First, we extend U so that it is defined on an open interval $I \supseteq X$ rather than just the chosen points, X, where $I \subset (0, \infty)$. For $z \in I - X$, let

$$U(z) := \sup_{x \in X} \{u(\hat{y}(x), z) - q(x)\}.$$

Thus, U is specified by prescribing optimal choices for all non-chosen types, and every point in X lies in the interior of I.

Lemma 3. *$U(x)$ is differentiable at every chosen $x \in X$.*

[4]That is, we assume that the set of chosen bundles is closed, so that every $x \in X$ has an associated pair $(\hat{y}(x), q(x))$ in the direct mechanism. This assumption is inessential, but simplifies the statement of some results.

Proof. Fix $x \in X$, and $\hat{y}(x)$. Consider the payoff of the consumer in the second period, $U(x+\delta)$ — this is well defined for δ sufficiently small since U is defined on the open interval I. Since $x+\delta$ can choose the contract for type x,

$$U(x+\delta) \geq u(\hat{y}(x), x+\delta) - q(x)$$

Thus, for $\delta > 0$

$$\frac{U(x+\delta) - U(x)}{\delta} \geq \frac{u(\hat{y}(x), x+\delta) - u(\hat{y}(x), x)}{\delta}.$$

The above inequality implies

$$D_+U(x) := \liminf_{\delta \to 0+} \frac{U(x+\delta) - U(x)}{\delta} \geq u_2(\hat{y}(x), x). \tag{1}$$

Since the inequality for $\delta < 0$ has a reversed sign, this yields

$$D^-U(x) := \limsup_{\delta \to 0-} \frac{U(x+\delta) - U(x)}{\delta} \leq u_2(\hat{y}(x), x). \tag{2}$$

Now, the total payoff of seller 1 and consumer, $\Pi(x)$, equals

$$\Pi(x) = U(x) - kx.$$

Let $D^+U(x) := \limsup_{\delta \to 0+} \frac{U(x+\delta)-U(x)}{\delta}$ and $D_-U(x) := \liminf_{\delta \to 0-} \frac{U(x+\delta)-U(x)}{\delta}$.
If $x \in X$, then, since x is chosen, it must maximize $\Pi(x)$, and necessary conditions are

$$\Pi^+(x) = D^+U(x) - k \leq 0,$$

$$\Pi^-(x) = D_-U(x) - k \geq 0.$$

These inequalities imply $D^+U(x) \leq D_-U(x)$. In conjunction with the inequalities (1) and (2), this implies that for any $x \in X$,

$$D^+U(x) = D_+U(x) = D^-U(x) = D_-U(x) = u_2(\hat{y}(x), x).$$

□

Remark 4. *The property that U is differentiable on X, the set of types that are chosen in equilibrium, follows from the endogeneity of types. With exogenous types, it is well known that U need not be everywhere differentiable. Indeed, this observation is more general than the specific context of our model.*

It is standard in mechanism design that single-crossing and incentive compatibility implies weak monotonicity. However, lemma 3 allows a stronger result.

Lemma 5. *$\hat{y}(x)$ must satisfy*

$$u_2(\hat{y}(x), x) = k.$$

Moreover, $\hat{y}(x)$ is strictly decreasing (resp. increasing) in x if u is submodular (resp. supermodular).

Proof. Since U is differentiable at $x \in X$, if x maximizes $\Pi(.)$, it must satisfy

$$\Pi'(x) = u_2(\hat{y}(x), x) - k = 0.$$

Consider a case of submodular utility — i.e., $u_{12} < 0$. If $x > x'$ then $\hat{y}(x)$ must be strictly less than $\hat{y}(x')$, or otherwise the expression for $\Pi'(.)$ above will be strictly negative. Similarly, in the case of supermodular utility — i.e., if $u_{12} > 0$, $\hat{y}(x)$ must be strictly greater than $\hat{y}(x')$. □

Let $\bar{x}$ denote the minimal element in X and $\bar{x}$ the maximal element. The following lemma shows that if individual rationality is satisfied for type $\bar{x}$ in the submodular case, then it is satisfied for every other type — although familiar, the result is not immediate since the outside option $u(0, x)$ is type dependent. Similar observation is made for to the case of supermodular utility.

Lemma 6. *If u is submodular, $U(x) - u(0, x) \geq U(\bar{x}) - u(0, \bar{x})$ for all $x \in X$. Moreover, under any profit maximizing second period contract, $U(\bar{x}) = u(0, \bar{x})$, and the individual rationality constraint binds for type $\bar{x}$. if u is supermodular, the individual rationality constraint binds for type $\underline{x}$.*

Proof. For $x < \bar{x}$, since type x can pretend to be $\bar{x}$, incentive compatibility implies that

$$U(x) \geq U(\bar{x}) + u(\hat{y}(\bar{x}), x) - u(\hat{y}(\bar{x}), \bar{x}).$$

Since $U(\bar{x}) \geq u(0, \bar{x})$,

$$U(x) - u(0, x) \geq [u(0, \bar{x}) - u(0, x)] - [u(\hat{y}(\bar{x}), \bar{x}) - u(\hat{y}(\bar{x}), x)], \tag{3}$$

which is non-negative since u is submodular and $\hat{y}(\bar{x}) \geq 0$.

If $U(\bar{x}) > u(0, \bar{x})$, then an mechanism $(\hat{y}(x), q(x))$ cannot be profit maximizing, since a uniform reduction in payoffs $U(x)$ by $U(\bar{x}) - u(0, \bar{x})$, achieved by raising $q(x)$ by the same amount, preserves incentive compatibility and increases profits. To obtain the same resulf for the case of supermodular u, replace $\bar{x}$ by $\underline{x}$ in the above argument. □

The following two lemmas identify $\underline{x}$ and $\bar{x}$. Recall that one of the bounds is identified using the fact that the highest type's consumption in the second period is efficient. In order to identify the other bound, we consider possible deviations by seller 1 and establish that the lowest type has to consume zero in the second period.

Lemma 7. *If utility is submodular, $\bar{x}$ equals the value of x that maximizes $u(0,x) - kx$ — i.e., $\bar{x} = x^o$, and $\hat{y}(\bar{x}) = 0$. For the case of supermodular u, $\underline{x}$ equals the value of x that maximizes $u(0,x) - kx$ — i.e., $\underline{x} = x^o$, and $\hat{y}(\underline{x}) = 0$.*

Proof. If u is submodular, since the second period participation constraint binds for the highest value of x that is offered by seller 1 and accepted by the consumer, the consumer is indifferent between $\hat{y}(\bar{x})$ and zero in the second period. By Lemma 1, there exists a profitable deviation for seller 1 unless $\hat{y}(\bar{x}) = 0$. Then, Lemma 5 implies that $\bar{x}$ must equal the value of x that maximizes

$$u(0,x) - kx,$$

or $\bar{x} = x^o$. The proof for the case of supermodular utility is identical. □

Lemma 8. *If the utility is submodular, then $\underline{x} = x^*$; and if it is supermodular, then $\bar{x} = x^*$. In either case, $\hat{y}(x^*) = y^*$.*

Proof. Recall that $y^*(x)$ denotes the first best second period quantity conditional on any level of the first period consumption x. Suppose that u is submodular. On one hand, since there is no distortion at the top in the second period screening problem, seller 2 must offer $y^*(\underline{x})$ to the consumer who consumed $\underline{x}$ in the first period. On the other hand, Lemma 5 establishes that $\underline{x}$ must satisfy

$$u_2(\hat{y}(\underline{x}), \underline{x}) = k.$$

These two conditions imply

$$u_2(y^*(\underline{x}), \underline{x}) = k,$$

which means that $(\underline{x}, y^*(\underline{x}))$ satisfies the conditions for the first best allocation. The first best allocation is unique, therefore $\underline{x} = x^*$. When u is supermodular, the "top" corresponds to $\bar{x}$, and the rest of the argument is the same. □

To summarize, the characterization in the above lemmata imply that $\bar{x} = x^o$ and $\underline{x} = x^*$ in the case of submodular utility. When u is supermodular, $\bar{x} = x^*$ and $\underline{x} = x^o$. Now we complete the description of the equilibrium — we find the distribution of the first period consumption F and the prices.

Theorem 9. *There exists an equilibrium in which*

1. *Seller 1 offers a two-part tariff. The entree fee equals to the seller 1's value added in the socially efficient consumption stream:*

$$u(y^*, x^*) - kx^* - u(y^*, 0).$$

The per-unit price equals to the marginal cost k.

2. *Seller 2 offers a menu that includes every bundle in $[0, y^*]$. The bundles in this menu are indexed by the first period consumption x. The price of a bundle $\hat{y}(x)$ is*

$$q(x) = u(\hat{y}(x), x) - kx - [u(0, \bar{x}) - k\bar{x}]$$

3. *In the first period, the consumer randomly chooses the bundle according to a distribution F. In the second period, he chooses a consumption $\hat{y}(x)$ where x is his first period consumption.*

If u is submodular, the support of the distribution F is $[x^*, x^o]$ and

$$F(x) = \exp\left[\int_x^{\bar{x}} \frac{u_{21}(\hat{y}(z), z)}{u_1(\hat{y}(z), z) - k} dz\right].$$

If u is supermodular, the support of the distribution F is $[x^o, x^*]$ and

$$F(x) = 1 - \exp\left[\int_{\underline{x}}^{x} \frac{u_{21}(\hat{y}(z), z)}{k - u_1(\hat{y}(z), z)} dz\right].$$

Proof. We verify that the proposed strategy profile is an equilibrium. For simplicity we focus on the case of submodular utility (the argument is very similar when u is supermodular).

The outside option of the consumer in the second period is $u(0, x)$. If the consumer consumes $\bar{x}$ in the first period, his continuation utility in the second period is $u(0, \bar{x})$, therefore

$$U(x) = u(0, \bar{x}) - \int_x^{\bar{x}} u_2(\hat{y}(z), z) dz.$$

The price charged by the seller 2 for the bundle $\hat{y}(x)$ is

$$q(x) = u(\hat{y}(x), x) - u(0, \bar{x}) + \int\limits_{x}^{\bar{x}} u_2(\hat{y}(z), z)dz, \tag{4}$$

and, hence, the expected profit for this seller is

$$\int\limits_{\underline{x}}^{\bar{x}} \left[u(\hat{y}(x), x)f(x) + u_2(\hat{y}(x), x)F(x) - k\hat{y}(x)f(x)\right] dx - u(0, \bar{x}).$$

Maximizing pointwise, we obtain that the allocation rule $\hat{y}(x)$ must satisfy the first order condition

$$u_1(\hat{y}(x), x)f(x) + u_{21}(\hat{y}(x), x)F(x) - kf(x) = 0. \tag{5}$$

Lemma 10. *If $\hat{y}(x)$ is decreasing and u is submodular (or If $\hat{y}(x)$ is increasing and u is supermodular), equation (4) implies IC in the second period, which is*

$$u(\hat{y}(t), x) - q(t) \leq u(\hat{y}(x), x) - q(x)$$

for all x, t.

Proof. Consider $x > t$. Since $\hat{y}(x)$ is decreasing and $u_{21} < 0$ (or, alternatively, $\hat{y}(x)$ is increasing and $u_{21} > 0$)

$$\begin{aligned}
&q(t) - q(x) = u(\hat{y}(t), t) - u(\hat{y}(x), x) + \int\limits_{t}^{x} u_2(\hat{y}(z), z)dz \geq \\
&u(\hat{y}(t), t) - u(\hat{y}(x), x) + \int\limits_{t}^{x} u_2(\hat{y}(t), z)dz = \\
&u(\hat{y}(t), t) - u(\hat{y}(x), x) + u(\hat{y}(t), x) - u(\hat{y}(t), t) = \\
&u(\hat{y}(t), x) - u(\hat{y}(x), x).
\end{aligned}$$

The case with $x < t$ is identical. □

Seller 1 makes the consumer indifferent between the inside and the outside options, therefore, he charges a price for amount x that equals

$$p(x) = u(\hat{y}(\underline{x}), \underline{x}) - u(\hat{y}(\underline{x}), 0) + \int_{\underline{x}}^{x} u_2(\hat{y}(z), z)dz.$$

In particular, the price for the bundle $\underline{x}$ equals

$$p(\underline{x}) = u(\hat{y}(\underline{x}), \underline{x}) - u(\hat{y}(\underline{x}), 0).$$

Seller 1's profit from selling a bundle x has to be independent of x, hence

$$u_2(\hat{y}(x), x) - k = 0. \tag{6}$$

Equations (6) and (5) pin down unknown functions F and $\hat{y}$.

Lemma 11. *Suppose that F and $\hat{y}$ solve equations (6) and (5). Then F is a c.d.f. and $\hat{y}$ is strictly decreasing (strictly increasing resp.) whenever u is submodular (supermodular resp.).*

Proof. By taking a derivative of equation (6) with respect to x we get

$$\hat{y}'(x) = -\frac{u_{22}(\hat{y}(x), x)}{u_{21}(\hat{y}(x), x)}$$

therefore $sign(\hat{y}'(x)) = sign(u_{21}(\hat{y}(x), x))$.

The solution to equation (5) is

$$\ln F(x) = \int_{x}^{\bar{x}} \frac{u_{21}(\hat{y}(z), z)}{u_1(\hat{y}(z), z) - k} dz \tag{7}$$

This solution is increasing in x if $u_1(\hat{y}(x), x) \geq k$ for all $x \in [\underline{x}, \bar{x}]$. Lower bound $\underline{x}$ solves $u_1(\hat{y}(\underline{x}), \underline{x}) = k$ since $F(\underline{x}) = 0$. Moreover, since both u and v are strictly concave

$$\begin{aligned}
\frac{d}{dx} u_1(\hat{y}(x), x) &= u_{21}(\hat{y}(x), x) + u_{11}(\hat{y}(x), x)\hat{y}'(x) \\
&= u_{21}(\hat{y}(x), x) - \frac{u_{22}(\hat{y}(x), x)}{u_{21}(\hat{y}(x), x)} u_{11}(\hat{y}(x), x) \\
&= \frac{u_{11}(\hat{y}(x), x)u_{22}(\hat{y}(x), x) - (u_{21}(\hat{y}(x), x))^2}{-u_{21}(\hat{y}(x), x)} > 0.
\end{aligned}$$

Therefore, $u_1(\hat{y}(x), x)$ is strictly increasing in x and $u_1(\hat{y}(x), x) \geq k$ for all $x \in [\underline{x}, \bar{x}]$. To summarize, the solution $F(x)$ is an increasing function, $F(\underline{x}) = 0$ and $F(\bar{x}) = 1$, therefore $F(x)$ is a c.d.f.[5] □

□

4.3 Uniqueness

All equilibria in this model have several common features. If the utility is supermodular, the equilibrium described in Theorem 9 is unique.[6] If the utility is submodular, there is a continuum of equilibria, and any of them differ from each other only in two ways:

(i) how the first period surplus is divided between seller 1 and the consumer; and

(ii) which items are offered by sellers, but never chosen by the consumer on the equilibrium path.

This multiplicity arises because seller 2 can add items that are larger than y^* to his menu and price any quantity above y^* at the marginal cost. Every such item (q, y) satisfies $y = y^* + \delta$ for some $\delta > 0$ and $q = q(y^*) + k\delta$. Note, that this items will not be chosen on equilibrium path. However, if the consumer decides not to consume in the first period, these items may potentially become more attractive than $(q(y^*), y^*)$ because $u(y^*, 0) > k$. Therefore, the value of the first-period outside option is weakly increasing in the variety of these items offered in the second period. Essentially, by offering these items, seller 2 reallocates the first-period surplus from seller 1 to the consumer.

To formalize these ideas we characterize the objects that are invariant across all equilibra. We focus on the submodular case — the results below apply equally to both cases. Fix an equilibrium σ of the game. Let V_σ denote the consumer's ex ante utility in this equilibrium. Let $\pi_{1,\sigma}$ denote the expected profit of seller 1 in this equilibrium, and let $\Pi_\sigma = V_\sigma + \pi_{1,\sigma}$ denote the sum of payoffs of the consumer and seller 1. Also, let X_σ denote the set of the first-period consumptions chosen in equilibrium σ, and $U_\sigma(x)$ denote the information rent of the consumer after choosing x in the first period

[5] Strictly speaking, the ODE (5) has a solution (7) on $(\underline{x}, \bar{x}]$, but we can continuously extend it to $\underline{x}$ with value of $\lim_{x \to \underline{x}+0} \exp\left[\int_x^{\bar{x}} \frac{u_{21}(\hat{y}(z), z)}{u_1(\hat{y}(z), z) - k} dz\right]$.

[6] Strictly speaking, it is possible to construct an equilibrium in which seller 1 randomizes and the consumer plays a pure strategy, but the distribution of consumption, prices and players' payoffs will be exactly the same as in the equilibrium characterized in Theorem 9.

of equilibrium σ. Let $\tilde{\sigma}$ denote the specific equilibrium constructed in the previous section, the support of which is the largest possible set, $X_{\tilde{\sigma}} = [x^*, x^o]$. Note, that by Lemmas 7 and 8, $X_\sigma \subset X_{\tilde{\sigma}}$.

Lemma 12. *For any equilibrium $\sigma : \Pi_\sigma = \Pi_{\tilde{\sigma}}$ and for any $x \in X_\sigma : U_\sigma(x) = U_{\tilde{\sigma}}(x)$.*

Proof. By Lemma 7 the maximal quantity offered and chosen in period one equals x^o in any equilibrium, and the informational rent that accrues to the consumer is zero in this case. Since x^o is in the support of every equilibrium, for any equilibrium σ

$$\Pi_\sigma = u(0, x^o) - kx^o = \Pi_{\tilde{\sigma}}.$$

For all $x^* \in X_\sigma$ the following holds in equilibrium

$$\Pi_{\tilde{\sigma}} = U_\sigma(x) - kx,$$

therefore $U_\sigma(x) = U_{\tilde{\sigma}}(x)$. □

In the light of this proposition, we write Π and $U(x)$ for the payoffs that arise in *any* equilibrium. Let F denote the c.d.f. associated with $\tilde{\sigma}$, as defined in the previous section, and let f denote the associated density.

Theorem 13. *1. If u is supermodular then the equilibrium is essentially unique.*

2. If u is submodular, then there is a continuum of equilibria. In every equilibrium,

- *(a) the distribution of the first-period consumption is F with the support on $[x^*, x^o]$;*
- *(b) the items of the second-period menu that are chosen on equilibrium path are $\{(q(x), \hat{y}(x))\}_{x \in [x^*, x^o]}$; and*
- *(c) the sum of the equilibrium payoffs of seller 1 and the consumer is Π.*

Proof. The only difference between the cases of sub- and supermodular utilities is the effect of second-period items that are not chosen by the consumer on equilibrium path. In the case of supermodular utility, these items do not affect the value of the first-period outside option. Given this observation, in this proof, we focus on the case of submodular utility.

Our proof hinges on two facts that have been established. First, for any equilibrium σ with support $X_\sigma : U_\sigma(x) = U(x)$, and second, $\hat{y}_\sigma(x)$, the bundle consumed in the second period in equilibrium σ by the consumer who consumed x in the past, is uniquely determined and coincides with that under $\tilde{\sigma} : \hat{y}(x)$. In other words, the payoff and the allocation for any chosen type is the same across all equilibria.

Let G denote the c.d.f. corresponding to σ, and let $M \subseteq X_\sigma$ denote mass points. Let $\mathcal{G}$ be a collection of maximal open intervals in $[x^*, x^o] \setminus X_\sigma$ that denote the gaps in X_σ. Hellwig (2010) characterizes the optimal allocation rule in a screening problem when the distribution of types includes mass points as well as intervals where the distribution is absolutely continuous with respect to Lebesgue measure, and his arguments also apply here. Let (a, b) be the "largest interval" in $\mathcal{G}$: i.e. for any other interval in the collection, say (c, d), $a > d$. Let m denote the largest value in M. Suppose that $m < a$, so that the first gap is larger than the first mass point. Now for every $x \in [b, x^o]$, $G(x) = F(x)$, since the necessary optimality condition for seller 2 coincides with the differential equation (5) and the initial condition is $F(x^o) = G(x^o) = 1$. In the equilibrium $\tilde{\sigma}$, the payoff for type a continues to be given by the differential equation, i.e.

$$U(a) = U(b) - \int_a^b [u_2(\hat{y}(x), x)dx.$$

In equilibrium σ, the binding incentive constraint is more slack, since it is given by the payoff that type b can get by pretending to be type a, and therefore

$$U_\sigma(a) = U(b) + [u(\hat{y}(b), a) - u(\hat{y}(b), b)]$$

Since $\hat{y}(x)$ is strictly decreasing, $U(a) > U_\sigma(a)$ which is a contradiction because Lemma 12 implies $U(a) = U_\sigma(a)$. Therefore, there can be no gap in $[m, x^o]$.

Now suppose that $m \geq b$, so that the largest mass point is greater than the the largest gap. $\hat{y}(m)$ must equal the same value in σ as it does in $\tilde{\sigma}$, and, therefore, must satisfy the first order condition for optimality. Since $\forall x \geq m : F(x) = G(x)$, $\hat{y}(m)$ will not be optimal for seller 2 if $G(m-0) < F(m-0)$. Hence $\hat{y}(m)$ cannot be optimal if there is a mass point at m. □

4.4 How does unobservability affect payoffs?

How does the consumers' privacy affect the payoffs of all participants? In order to answer this question, we compare the equilibrium in the benchmark model, in which the past is observable by seller 2, to equilibria characterized in Theorem 13. We find that, when the past becomes unobservable,

(i) the social welfare, the consumer's utility and the seller 2's profit increase;

(ii) the first-period surplus does not change; and

(iii) the seller 1's profit decreases.

Intuitively, the social welfare increases because the amount of distortion introduced by optimal choices of sellers decreases — with a positive probability they sell bundles that are close to the first-best stream of consumption. The aggregate effect on sellers' profits results from two conflicting forces: on the one hand, consumer's private information limits the ability of seller 2 to extract surplus, but, on the other hand, seller 2's ignorance of seller 1's menu protects him from seller 1's expansionist actions of selling large quantities. The latter effect resembles Bagwell's paradox (see Bagwell, 1995).

If the past is not observed by seller 2, the consumer can choose the outside option in period 1 without seller 2 reacting by an increase of the prices in period 2. This increases the value of the first-period outside option compared to the benchmark model with observable consumption.

Finally, the first-period surplus is the same as in the case of observable past consumption because there is at least one type of the consumer that has no information rent in the future — she takes outside option in the second period. Recall, that if the past is observable, in the first period, the consumer is treated as if she does not buy in the second period.

Recall, that Π is the first-period surplus when the past is unobservable; Π^o — when the past is observable. Let π_i denote the profit of seller $i \in 1, 2$, V — the equilibrium payoff of the consumer and W — social welfare. The superscript o denotes the benchmark case in which the past is observable, and the subscript σ — associates a variable with equilibrium σ.

Theorem 14. *If u is supermodular, then*

(i) $\Pi = V + \pi_1 = V^o + \pi_1^o$;

(ii) $\pi_1 < \pi_1^o$ *and* $V > V^o$; *and*

(iii) $W - W^o = \pi_2 - \pi_2^o > 0$.

If u is submodular, then for any equilibrium σ

(i) $\Pi = V_\sigma + \pi_{1,\sigma} = V^o + \pi_1^o$;

(ii) $\pi_{1,\sigma} \leq \pi_{1,\tilde{\sigma}} < \pi_1^o$ *and* $V_\sigma \geq V_{\tilde{\sigma}} > V^o$; *and*

(iii) $W_{\tilde{\sigma}} - W^o = W_\sigma - W^o = \pi_{2,\sigma} - \pi_2^o > 0$.

Proof. Note that the first-period surplus, Π, can be evaluated at any point in the support. In particular, at x^o, the consumer takes his outside option in the second period, and so

$$\Pi = u(0, x^o) - kx^o.$$

This is *identical* with the total payoff of the seller 1 and the consumer when consumption is observable — although the consumer purchases $y^o > 0$, seller 2 appropriates the the difference $u(y^o, x^o) - u(0, x^o)$, and hence the consumer's continuation payoff is $u(0, x^o)$. We turn to the distribution of the total payoff between the two parties in the two cases. In the observable case, the consumer's payoff equals

$$V^o := u(0, 0).$$

In the unobservable case, the results now differ depending on whether u is supermodular or submodular. So we consider these in turn.

When u is supermodular, the consumer who chooses the outside option in the first period, chooses the item $\hat{y}(x^o) = 0$ in the second period, and therefore gets a total payoff

$$V = u(0, 0).$$

This is exactly equal to V^o, and hence unobservability has no distributional effect on the first period payoffs when u is supermodular.

When u is submodular, there is a continuum of equilibria that differ by the value of the outside option in the first period. Consider the equilibrium with the smallest such value — i.e., the equilibrium $\tilde{\sigma}$ characterized in Theorem 9. In this equilibrium, if the consumer chooses the outside option in the first period, she buys $\hat{y}(x^*) = y^*$ in the second period and, therefore, gets a total payoff

$$\underline{V} := V_{\tilde{\sigma}} = [u(y^*, 0) - u(y^*, x^*)] + u(0, x^*).$$

The difference in payoffs is

$$\underline{V} - V^o = [u(0, x^*) - u(0, 0)] - [u(y^*, x^*) - u(y^*, 0)] = \pi_1^o - \overline{\pi}_1 > 0,$$

where π_1^o denotes seller 1's profits in the observable case. The second equality in the above follows since the total payoff Π is equal in the two cases. The strict inequality arises since u is strictly submodular.

Now consider the equilibrium with the largest value of the first-period outside option. Using the same argument we obtain that

$$\overline{V} := [u(y^*(0), 0) - u(y^*, x^*)] - k(y^*(0) - y^*) + u(0, x^*) > \underline{V}.$$

and $\overline{\pi}_1 > \underline{\pi}_1$. For every $V \in [\underline{V}, \overline{V}]$, there exists an equilibrium in which the consumer's payoff is V and seller 1's profit is $\Pi - V$.

We conclude that, in any equilibrium, the consumer is strictly better off when consumption is unobservable, and seller 1 is strictly worse off to exactly the same extent.

Since Π is the same across all equilibria including the benchmark case of the observable past, the gain (loss) of seller 2 from unobservability of past consumption equals to the increase (decrease) of the social welfare. Morever, this gain (loss) is the same for all equilibria because the equilibrium distribution of consumption is the same.

To see that $\pi_2 > \pi_2^o$, note that if seller 2 offers a single item (q^o, y^o), it will be accepted with probability 1 because every consumer's type is better (in terms of marginal willingness to pay) than x^o. The fact that such a menu is not offered implies that $\pi_2 > \pi_2^o$. □

The equilibrium consumption is distorted by the intertemporal competition between the sellers in an unusual way. If the utility is submodular, the consumer always over-consumes in the first period and under-consumes in the second [cite other models here]. The realized social welfare is monotone in the first period consumption.

Remark 15. *Equilibrium social welfare conditional on first period consumption x is decreasing in x when u is submodular, and increasing when u is supermodular.*

Proof. Equilibrium social welfare conditional on first period consumption x is

$$W(x) = v(x) + u(\hat{y}(x), x) - k(x + \hat{y}(x))$$

Taking a derivative, we obtain

$$\begin{aligned} W'(x) &= v'(x) + u_2(\hat{y}(x), x) - k + \hat{y}'(x) u_1(\hat{y}(x), x) - k\hat{y}'(x) \\ &= \hat{y}'(x) \left[u_1(\hat{y}(x), x) - k \right], \end{aligned}$$

where the second line follows from the first period first order condition, equation (6). Since the second period consumption is always distorted, the term in square brackets is always positive, and hence W is increasing in x when $\hat{y}$ is (the supermodular case), and decreasing in x when $\hat{y}$ is decreasing in x, as in the submodular case. □

4.5 An example

In this section we consider a numerical example of our model. Suppose that the utility of the agent is

$$u(y, x) = -3x^2 - 3y^2 + axy + 8x + 8y$$

Table 1: Numerical example

	$a = -1$	$a = -3$
x^*	1	0.78
x^o	1.17	1.17
y^*	1	0.78
y^o	0.97	0.58
$W^\star$	7	5.44
W^o	6.92	5.10
W	6.99	5.39
π_1^o	4.08	4.08
π_2^o	2.84	1.02
π_1	[2.92, 3.00]	[1.36, 1.81]
π_2	2.92	1.31
V^o	0	0
V	[1.08, 1.16]	[2.27, 2.72]

Note: Superscript * denotes variables for the first best case and superscript ^{o} denotes variables for the benchmark model with observable consumption.

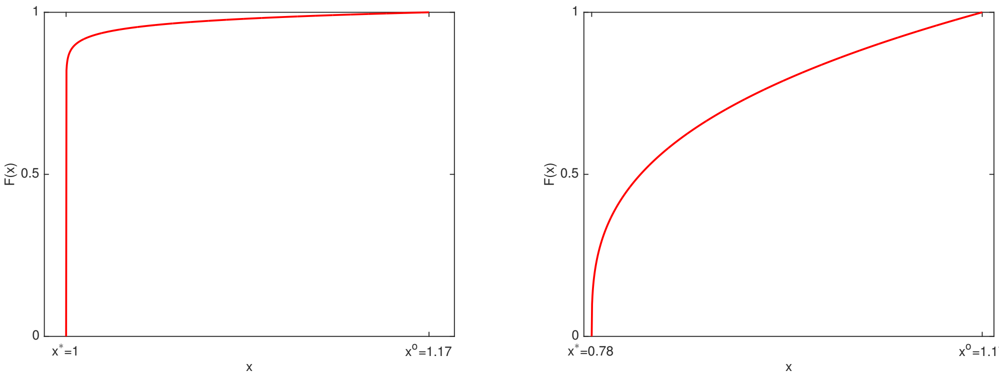
The parameter $a = u_{21}(y, x)$ is a measure of substitutability of the past and current consumption.

The variables of interest for this example are presented in Table 1. The equilibrium distributions of first period consumption for the two cases, $a = -1$ and $a = -3$ are given on Figure 1.

In these two cases, the equilibrium distribution is skewed to the left: most of the consumers consume an amount close to the socially efficient one. This is also reflected in the fact that efficiency loss in equilibrium, $W^* - W$, is small compared to the one in benchmark model with observable consumption, $W^* - W^o$.

The distribution of the social welfare is also interesting: consumers and the second period seller obtain higher payoffs when past consumption is unobservable compared to the case when past consumption is observable. The profit comparison for the first period seller is the opposite of that: this seller's profit is reduced by unobservable past consumption.

According to the results in Section 4.4, the first seller's loss is consumer's gain and social welfare's increase is fully absorbed by the second seller. This result is reflected



(a) $a = -1$;

(b) $a = -3$;

Figure 1: Distribution of first period consumption.

in the numbers obtained for this example. Notice that the gain from unobservability of past consumption for seller 2 is much smaller then the gain for the consumer.

5 Concluding remarks

We study optimal nonlinear pricing in the presence of inter-temporal substitutability and complementarity. We show that, even if consumers are ex ante identical, the equilibrium menus offered by sellers feature a large variety of bundle sizes paired with quantity discounts. These offerings give rise to persistent endogenous taste heterogeneity across consumers.

Unlike classic models of nonlinear pricing — e.g., Maskin and Riley (1984) — in which sellers face exogenously heterogeneous population of buyers, our model features identical consumers. We derive testable relationship between the price dispersion and the degree of inter-temporal substitutability or complementarity. In our model, if past choices are not observed by the sellers, the consumers necessarily retain some surplus in the form of information rent. The informational content of the past choices depends on the degree of inter-temporal substitutability or complementarity and it affects the optimal menus offered by the sellers.

The findings are consistent with supermarkets and convenience stores offering a large variety of bundle sizes for the same product.

References

Aghion, Philippe and Patrick Bolton, "Contracts as a Barrier to Entry," *The American Economic Review*, 1987, pp. 388–401.

Ariga, Kenn, Kenji Matsui, and Makoto Watanabe, "Hot and spicy: ups and downs on the price floor and ceiling at Japanese supermarkets," Mimeo, The Australian National University 2001.

Bagwell, Kyle, "Commitment and Observability in Games," *Games and Economic Behavior*, 1995, *8* (2), 271–280.

Bernheim, B Douglas and Michael D Whinston, "Menu auctions, resource allocation, and economic influence," *The Quarterly Journal of Economics*, 1986, pp. 1–31.

Calzolari, Giacomo and Alessandro Pavan, "On the optimality of privacy in sequential contracting," *Journal of Economic Theory*, 2006, *130* (1), 168–204.

Cole, Harold L. and Narayana R. Kocherlakota, "Dynamic Games with Hidden Actions and Hidden States," *Journal of Economic Theory*, May 2001, *98* (1), 114–126.

Diamond, Peter A and Eric Maskin, "An equilibrium analysis of search and breach of contract, I: Steady states," *The Bell Journal of Economics*, 1979, pp. 282–316.

Fox, Edward J, Alan L Montgomery, and Leonard M Lodish, "Consumer shopping and spending across retail formats," *The Journal of Business*, 2004, *77* (S2), S25–S60.

Fudenberg, Drew and Jean Tirole, "Moral hazard and renegotiation in agency contracts," *Econometrica: Journal of the Econometric Society*, 1990, pp. 1279–1319.

González, Patrick, "Investment and screening under asymmetric endogenous information," *RAND Journal of Economics*, 2004, pp. 502–519.

Hendel, Igal, Alessandro Lizzeri, and Nikita Roketskiy, "Nonlinear pricing of storable goods," *American Economic Journal: Microeconomics*, 2014, *6* (3), 1–34.

_ **and Aviv Nevo**, "Measuring the Implications of Sales and Consumer Inventory Behavior," *Econometrica*, November 2006, *74* (6), 1637–1673.

_ **and** _ , "Sales and consumer inventory," *RAND Journal of Economics*, September 2006, *37* (3), 543–561.

Holmstrom, Bengt and Paul R. Milgrom, "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, 1987, *55*, 303–328.

Hong, Pilky, R Preston McAfee, and Ashish Nayyar, "Equilibrium price dispersion with consumer inventories," *Journal of Economic Theory*, 2002, *105* (2), 503–517.

Ma, Ching-to Albert, "Adverse selection in dynamic moral hazard," *The Quarterly Journal of Economics*, 1991, pp. 255–275.

Martimort, David and Lars Stole, "The revelation and delegation principles in common agency games," *Econometrica*, 2002, *70* (4), 1659–1673.

Maskin, Eric and John Riley, "Monopoly with incomplete information," *Rand Journal of Economics*, 1984, *15*, 171–96.

Netzer, Nick and Florian Scheuer, "Competitive screening in insurance markets with endogenous wealth heterogeneity," *Economic Theory*, 2010, *44* (2), 187–211.

Thomassen, Øyvind, Howard Smith, Stephan Seiler, and Pasquale Schiraldi, "Multi-category competition and market power: a model of supermarket pricing," *American Economic Review*, 2017.

Villas-Boas, J. Miguel, "Dynamic Competition with Customer Recognition," *RAND Journal of Economics*, Winter 1999, *30* (4), 604–631.

_ , "Price Cycles in Markets with Customer Recognition," *RAND Journal of Economics*, Autumn 2004, *35* (3), 486–501.

_ , "Dynamic Competition with Experience Goods," *Journal of Economics & Management Strategy*, March 2006, *15* (1), 37–66.